# 1.1 Nice to Meet You!

Handshaking is a friendly gesture and an almost universal form of greeting. Almost everyone does it when he/she meets someone new. The situation is so intriguing and mathematically rich that there is even a theorem named for it. Do you often shake hands with others? Let's take a closer look.

## 1.1.1 Investigate: What Stays the Same? What Changes?

**Focus Question: What stays the same and what changes when different groups of people shake hands?**

1) Meet and greet other students in your class by shaking hands with each other. Record the total number of students whose hands you shook.

   For this activity, you are restricted to the following conditions:

   - ✓ Only one handshake is allowed between any two people.
   - ✓ No one may shake hands with themselves.
   - ✓ You don't need to shake hands with anyone if you don't want to.
   - ✓ All handshakes are between two people at a time.

   At your teacher's signal, report the total number of hands you shook.

## 1.1.2 Investigate: Handshake Impossible?

**Focus Question: What makes a given list of numbers of handshakes performed by each individual possible or impossible?**

For this activity, each person cannot shake hands with another person more than once, and no one can shake hands with himself or herself.

1) Form groups of 3 or 4. In your group, individually write a number of hands you would like to shake. Report this list to your teacher.

I would like to shake _____ hands.

2) As a group, decide if it is possible to perform all the handshakes with the desired numbers. If it is possible, find a way to demonstrate the procedure of handshakes. If not, explain why it is not possible.

3) As a group, determine all possible combinations of hands each person could shake based on the number of people in your group. At your teacher's signal, report your data.

4) Using all the lists of numbers of handshakes reported in your whole class, write at least one statement that you think is true about the lists of handshakes that worked or those that did not work. What do you think makes it true? Be prepared to share your statement(s) and justification.

5) Here is a list of hands that students would like to shake for several groups. Decide if it is possible to complete all the handshakes with the desired numbers. Explain.

   a) (2, 2, 1)

   b) (4, 3, 2, 1)

   c) (3, 1, 1, 1)

   d) (4, 3, 2, 2, 1)

   e) (3, 3, 1, 1, 0)

## 1.1.3 Explain: What Causes the Results?

**Focus Question: What causes the statements to be true?**

Here are a few statements that other students have made about a similar situation. You may have come up with these yourself in the previous tasks. Considering only handshake sequences that work, which statements are true according to your class data? Which statements are always true in general? What makes each statement true? Write up your reasoning for each of the statements.

1) The sum of all the hands shaken reported from all individuals is even.

2) The number of students who shook an odd number of hands is even.

3) The number of students who shook an even number of hands is even.

4) In order for the list of hands shaken to be possible, at least one number must repeat.

### 1.1.4 Extend: Handshake Impossible (Redux)

Come up with a step-by-step procedure that can be used to determine if it is possible to perform all the handshakes from a list of handshakes that individuals in a group would like to perform.

### 1.1.5 Reflect: How Does the Structure Help to Detect Errors?

Revisit the class data from task 1. Using your knowledge and reasoning from tasks 2 through 4, how can you tell whether or not the list could contain an error? Explain.

## 1.1.6 Investigate: Count Them Up

**Focus Question: How do we count the number of handshakes in each of the following scenarios? How can we be sure that we did not over-count or under-count?**

For the following questions, if two people are shaking hands with each other, we count that as one handshake.

1) Suppose everyone in the class stands in a single line and shakes hands with the people in front and behind them. How many handshakes would occur?

2) Suppose everyone in the class stands in a circle and shakes hands with the two people on either side of them. How many handshakes would occur?

3) Suppose all the students in the class stand in a circle, and the teacher stands in the middle of the circle. The teacher shakes hands with each student once and every student shakes hands once with the two students on either side of them. How many handshakes would occur?

4) Suppose every member of the class with a first name beginning with the letters A-M shakes hands with every member of the class with a first name beginning with the letters N-Z, and no member of either group shakes hands with any member of that same group. How many handshakes would occur?

5) Suppose everyone in the class shakes hands with everyone else exactly once. How many handshakes would occur?

# Nice to Meet You! Problem Set

## Check for Understanding

1) Revisit the handshaking situation in your class. Answer the following questions.
   a) What conditions were imposed in the situation?
   b) Regardless of the number of people in the group, what stays the same? What changes?
   c) Determine if each of the following statements is always true, sometimes true, or always false. Explain why.
      i. The number of students who shook an even number of hands is odd.
      ii. The number of students who shook an odd number of hands is odd.

2) During the handshaking activity in a class, Joe claimed: "The total number of hands each person reported shaking must add up to an even number." When asked to justify his statement, Joe said, "It is because there is an even number of people in the class." Is Joe's claim correct? Is his reason for the claim correct? Why or why not?

3) A group of five students shook hands. Only one handshake occurred between any two students. When asked to report the number of hands each person shook, the students reported 3, 3, 3, 3, and 2. How many handshakes took place?

4) A group of friends meet up at a party. Some of them greet each other with fist bumps. Determine if the following numbers of fist bumps are possible. If so, describe a procedure of the fist bumps. If not, explain why not.
   a) In a group of 3, the desired fist bump sequence is (1, 1, 1).
   b) In a group of 4, the desired fist bump sequence is (3, 3, 2, 2).
   c) In a group of 5, the desired fist bump sequence is (5, 2, 2, 1, 0).

5) Suppose a group of 20 mathematicians shake hands at a conference. Each person shakes hands with everyone else exactly once. How many handshakes are there?

## Repeated Reasoning

6) A group of 11 friends likes to chat with each other on the phone at night. Each phone call takes place between two people. One day, the group meets up at school and finds that the previous night, each of them had talked on the phone with 5 of their friends in this group. Jose exclaims, “That’s impossible!” Is he right?

7) A group of six friends met at a party. Some of them shook hands. Show that there must be two people who gave the same number of handshakes.

8) Two chess clubs have 15 members each. The club officers decided that each team member would play a total of 10 games, 5 of which were to be with different members of their own clubs and the other 5 games with different members outside of their own club. Is it possible? Why or why not?

9) Show that within any group with an odd number of people, there is at least one person who knows an even number of people.

10) In the table below you will see a list of people and the number of handshakes they wish to perform.

| | Round 1 | Round 2 | Round 3 | Round 4 | Round 5 | Round 6 |
|---|---|---|---|---|---|---|
| Andrew | 1 | 1 | 2 | 5 | 5 | 5 |
| Bernard | 2 | 2 | 2 | 1 | 2 | 4 |
| Cori | 2 | 2 | 2 | 1 | 2 | 3 |
| Devon | 0 | 1 | 2 | 1 | 1 | 2 |
| Elliot | 1 | 3 | 2 | 1 | 1 | 2 |
| Forest | 0 | 1 | 2 | 1 | 0 | 1 |

a) If no pairing can occur more than once, and no one can shake his/her own hand, which round(s) will occur without having anyone left with an unfulfilled handshake? Make a representation of how the round might occur.

b) How do you know that a given sequence will leave a hand unshaken? Think through your reasoning process, then do your best to write down your reasoning.

c) Create two new rounds where everyone's handshakes are fulfilled.

d) Create a new round that leaves one person with an unfulfilled handshake.

e) Create a new round that leaves more than one person with unfulfilled handshakes.

## Diving Deeper

11) Jack and his wife, Jill, invited three other married couples to their house for dinner. Various handshakes took place. No one shook his/her own hand or his/her spouse's hand, and no one shook hands with the same person more than once. After all the handshakes were over, Jack asked each guest and his wife (but of course doesn't ask himself), how many hands he/she had shaken. Surprisingly, each person gave a different answer. How many handshakes took place? How many hands did Jack shake? How many hands did Jill shake?

12) Four mixed couples (a man and a woman per couple) are needed for a tennis team. Five men and four women are available. The following criteria must be considered.

- ✓ Allan will not play with Fay or Helen;
- ✓ Ben will not play with Izzie;
- ✓ Fay, Gloria and Helen will not play with Earl;
- ✓ Derek will not play with Helen or Izzie; and
- ✓ Carl will only play with Gloria.

Can a team be put together under these conditions? If so, how?

13) Play the Havel-Hakimi Game to reinforce your understanding of handshake sequence at **http://jacquerie.github.io/hh/** or **https://tinyurl.com/havel-hakimi**. It can be addicting!