

1.1 Thai 21

- 1) We've all played board games and card games before. Some games you may have heard of have been placed into three categories below. Talk in your group about the differences between the games in different categories and the similarities of the games across the chart.

Can you list one more game you've played that would fall in each of these categories? What are some games you've played that might fall outside the chart, and why?

Tic-Tac-Toe	Poker	Candy Land
Connect Four	Monopoly	The Game of Life
Chess	Battleship	Snakes and Ladders
Checkers	Cluedo	Trouble/Sorry!

- 2) Your teacher will show you a short video from the reality show *Survivor*.

In your group, brainstorm a list of questions you have from watching the video!

- 3) Work together with your group to answer the questions your teacher has circled on the board! Use the space below to take notes if you need to.

Remember the rules of the game: We start with 21 flags. On each team's turn, they must take 1, 2, or 3 flags. The team that takes the last flag wins.

- 4) Suppose we change the game a little: We start with 21 flags. On each team's turn, they must take 1, 2, or 3 flags. The team that takes the last flag *loses*.

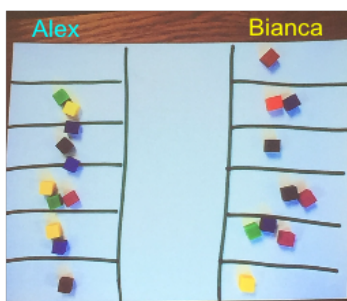
How do your answers to the questions the teacher has circled on the board change for this game?

- 5) How would you answer the questions the teacher has circled on the board, for the original *Thai 21* game? What are some problem-solving strategies you used in solving these problems?

Thai 21 – Problem Set

Check for Understanding

- You are playing Thai 21 with a classmate. The rule of the game is as follows.
Start with 21 flags. On each player's turn, they must take 1, 2, or 3 flags. The player that takes the last flag *wins*.
Consider the following scenarios.
 - Four flags remain and it is your turn. List the remaining moves for all possible ways the game could play out. Will you or your opponent win, assuming both of you know the winning strategy? Explain.
 - It's your turn and there are 9 flags remaining. How many flags should you take on your turn? Can you win the game? Why or why not?
 - It's your turn and there are 12 flags remaining. How many flags should you take on your turn? Can you win the game? Why or why not?
- Start with 21 flags. On each player's turn, they must take 1, 2, or 3 flags. The player that takes the last flag *loses*.
 - Four flags remain and it is your turn. Will you or your opponent win? Explain.
 - Five flags remain and it is your turn. Will you or your opponent win? Explain.
 - It's your turn and there are 9 flags remaining. How many flags should you take on your turn? Can you win the game? Why or why not?
 - It's your turn and there are 12 flags remaining. How many flags should you take on your turn? Can you win the game? Why or why not?
- While watching Alex and Bianca play a round of Thai 21 game during class, Randy pretended to be a games announcer and reported every move they made. For fun, he commented on whether each move helped or hurt the player's chance of winning. The entire game was recorded below. Help Randy write a play-by-play commentary on this game.



Repeated Reasoning

4. Start with 32 flags. On each player's turn, they must take 1, 2, or 3 flags. The player that takes the last flag wins.
 - a. Should you go first or second to guarantee yourself a win? What should your first move be?
 - b. What are all your remaining moves to guarantee yourself a win?
 - c. Why does this strategy guarantee you a win?

5. Start with 32 flags. On each player's turn, they must take 1, 2, 3, or 4 flags. The player that takes the last flag wins.
 - a. Should you go first or second to guarantee yourself a win? What should your first move be?
 - b. What are all your remaining moves to guarantee yourself a win?
 - c. Why does this strategy guarantee you a win?

6. Start with 32 flags. On each player's turn, they must take 1, 2, 3, or 4 flags. The player that takes the last flag loses.
 - a. Should you go first or second to guarantee yourself a win? What should your first move be?
 - b. What are all your remaining moves to guarantee yourself a win?
 - c. Why does this strategy guarantee you a win?

7. What is the winning strategy for each of the following variants of the game Thai 21? Be sure to include in your strategy:
 - Should you go first or second to guarantee yourself a win? What should your first move be?
 - What are all your remaining moves to ensure yourself a win?
 - Why does this strategy guarantee you a win? Consider why your opponent cannot win or why you cannot lose if you are playing a perfect game.
 - Provide lists, diagrams, arithmetic, equations, and/or graphs, etc., that illustrate your strategy.
 - a. We start with n flags. On each team's turn, they must take away 1, 2, 3, ..., or k flags. The team who takes the last flag wins.

- b. We start with n flags. On each team's turn, they must take away 1, 2, 3, ..., or k flags. The team who takes the last flag loses.

8. Abram and Zach played a game called Russian 12. Here are the rules of the game:

Start with 12 flags. Each player must take 1 or 2 flags on their turn. The player who takes the last flag or pair of flags wins.

When asked to explain their strategy, Abram said the following:

"Player should try to leave it in multiples of 4 (4, 8, or 12) or possibly even multiples of 2 (2, 4, 6, 8, 12). The player trying to win should try to keep control and take 2 flags in order to even it out because if a player took one flag in the beginning of the game, it's most likely the other player will win. Actually, I think I'd try to keep it at odd numbers like 1, 3, 5 in order to win the last flag."

Do you agree with Abram's strategy? Explain why or why not. How would you respond to Abram?

9. How are these games the same or different?

Game 1: We start with 21 flags. On each team's turn, they must take away 1, 2, or 3 flags. The team who takes the last flag *wins*.

Game 2: We start with 42 flags. On each team's turn, they must take away 2, 4, or 6 flags. The team who takes the last flag *wins*.

Diving Deeper

10. Compare and contrast the winning strategies of Thai 21 for when the last player wins vs. when the last player loses. What is it about the rules of the game that cause a player to be in a winning or losing position for each version of the game?
11. We start with 21 flags. On each team's turn, they must take away 1, 3, or 4 flags. The team who takes the last flag wins. What is the winning strategy for this game?
12. We start with 82 flags. On each team's turn, they must take away 1, 4, or 5 flags. The team who takes the last flag wins. What is the winning strategy for this game?
13. Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbara's turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. When it is Jenna's turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. If both players use their best strategy, who will win the game if the game starts with 2018 coins?